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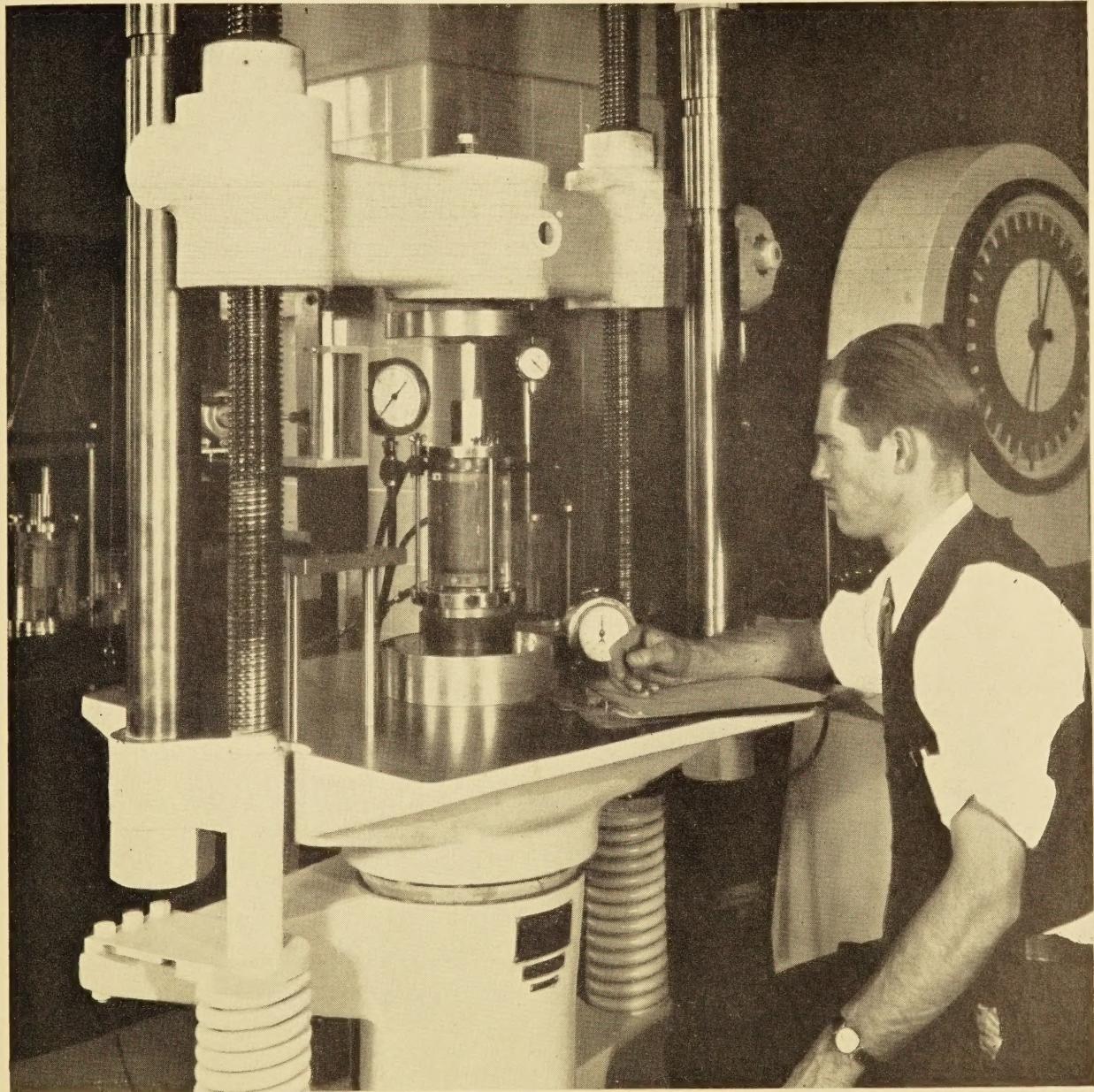
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LABORATORY DETERMINATION OF SOIL STABILITY

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The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to described conditions.

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THE SETTLEMENT OF EARTH EMBANKMENTS

BY THE DIVISION OF TESTS, PUBLIC ROADS ADMINISTRATION

Reported by L. A. PALMER, Associate Research Specialist, and E. S. BARBER, Junior Highway Engineer

ONE of the most important considerations in the design of highway embankments is the amount of settlement that may be expected. Methods of estimating settlement caused by soil consolidation due solely to loss of water from compressible foundation soils have already been described.¹ The present report is chiefly concerned with settlement resulting from lateral displacement of soil, designated as S_L , as distinguished from settlement caused by consolidation, designated as S_c .

The question of ultimate supporting power has been considered at length in three previous publications^{2 3 4} and the analyses are made with reference to ultimate fill loads that will cause the supporting earth to fail completely, that is, deform without any definite limit. Obviously, however, more information is required for the satisfactory design of embankments.

For example, it is entirely possible for a high fill to subside several feet due to displacement in the supporting soil but without the occurrence of failure in that supporting soil. Assumption in the design that a large factor of safety against ultimate failure will assure one that displacement by lateral yield will be a small quantity may not accomplish the desired result. Hogentogler and Allen⁵ have pointed out the fallacy of such an assumption and have proposed the use of values, c' and ϕ' , which are certain percentages of the unit cohesion c , and the angle of internal friction ϕ , respectively, which appear in formulas for computing the supporting power of the earth below the fill. This procedure is based on the use of the complete shearing stress-deformation relation instead of ultimate values.

PLASTIC YIELD ASSUMED INSTEAD OF ELASTIC DEFORMATION

In continuation of this study, it is proposed to make use of a principle presented by A. Nadai⁶ called the "stationary flow of a plastic mass." This principle leads to the development and use of formulas that are similar to the expressions for Hooke's law for elastic bodies but which differ from Hooke's law in that plastic-yield is assumed in the place of elastic deformation.

The basic principle involved in this report is a simple one and is that a certain earth movement, too small to be comparable to displacements characteristic of failure of the earth itself, may ruin a structure. More specifically, the fill load may be much too small

to cause failure of the supporting earth, yet it may be large enough to be disastrous to the highway. In brief, it is necessary to estimate S_L . For this purpose two things are needed: A ratio of stress to deformation, herein designated as C , the modulus of deformation, which has no reference to the nature of the deformation, whether it be elastic or plastic deformation or both; and a value of Poisson's ratio, μ .

Experimental work has indicated that the value of μ for compressible types of soil may vary from 0.35 for soils containing much air to 0.50 for soils that are saturated with water.

For the cases in which μ is less than $\frac{1}{2}$, it is very likely that the small soil samples undergo some degree of volume diminution owing to the escape of air or water during the stabilometer test. It cannot, however, be concluded that the same volume change occurring in such tests can take place as readily and quickly in large earth masses. Therefore, it has been customary to take μ as $\frac{1}{2}$ in computing S_L in large earth masses even though the laboratory value of μ is not this quantity, and this value of μ is used in computing S_L in this report.

Figure 1 illustrates the essential features of the laboratory stabilometer used in the study of stresses within a cylindrical soil sample. The stabilometer is often referred to as the "triaxial shear test device." Several types of this device and the principles governing their use have been described by Hogentogler and Barber.⁷

In stabilometer tests, cylindrical soil samples, encased in rubber sleeves, are compressed to complete failure by the application of vertical load. During the loading there may be no lateral pressure on the specimen (a simple compression test in this case) or a variable or a constant lateral pressure may be applied from start to finish of the test. The test may be made with or without porous stones at the flat ends of the cylindrical samples.

The vertical load is applied through the plunger by means of a hydraulic testing machine. At the beginning of a test, the head of the machine is lowered until contact is made with the plunger, as shown in the cover illustration, and the platen and head of the machine remain fixed in position until loading is begun. A definite fluid pressure is then applied to the sample, figure 1, through the inlet valve. Since the machine is fixed in position the lateral fluid pressure tends to push a saturated sample upward against the plunger with a variable force depending on the mobility of the sample. For a saturated soft soil, the vertical and lateral pressures may become equal under these initial conditions. When the sample contains much air the fluid pressure tends to compress the air or cause its escape thus shortening the sample without increasing its diameter.

¹ The Theory of Soil Consolidation and Testing of Foundation Soils, by L. A. Palmer and E. S. Barber. PUBLIC ROADS, vol. 18, No. 1, March 1937.

² Principles of Soil Mechanics Involved in Fill Construction, by L. A. Palmer and E. S. Barber. Proceedings of the Highway Research Board, Seventeenth Annual Meeting, Dec. 1937.

³ Principles of Soil Mechanics Involved in the Design of Retaining Walls and Bridge Abutments, by L. A. Palmer. PUBLIC ROADS, vol. 19, No. 10, Dec. 1938.

⁴ Design of a Fill Supported by Clay Underlaid by Rock, L. A. Palmer. PUBLIC ROADS, vol. 20, No. 8, Oct. 1939.

⁵ Important Considerations in Soil Mechanics, by C. A. Hogentogler and Harold Allen, Bulletin, American Society for Testing Materials, No. 94, Oct., 1938.

⁶ Plasticity (chapter 14, pp. 75 to 79, incl.), A. Nadai. Engineering Societies Monographs, McGraw-Hill Book Company, Inc., first edition, 1931.

⁷ Essential Features of Triaxial Shear Tests, by C. A. Hogentogler and E. S. Barber. PUBLIC ROADS, vol. 20, No. 7, Sept., 1939.

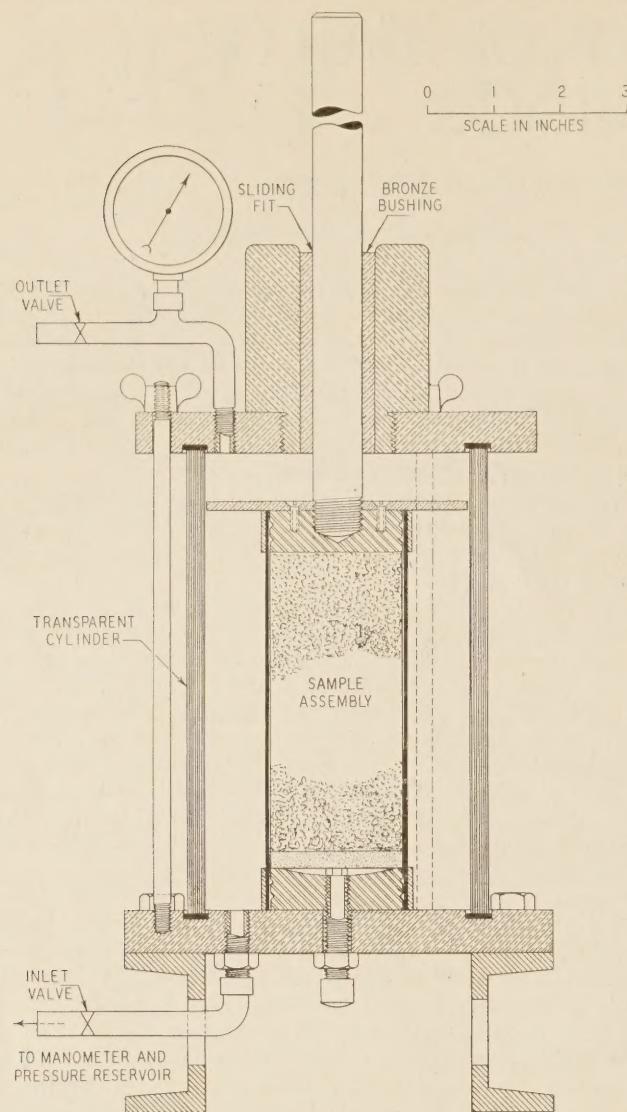


FIGURE 1.—STABILOMETER OF THE PLUNGER TYPE.

Vertical load is then increased by elevating the platen at a constant rate of 0.05 inch per minute. An automatic recording device gives the complete vertical load versus change in height curve for the entire test. Any change from the initial height, h , is designated as Δh .

For the plotted data shown in this paper, the soil cylinders were 1.95 inches in diameter and the height, h , was 4 inches. Porous stones, sometimes placed at the two flat ends of the sample, were not used in these tests.

MODULUS C DETERMINABLE FROM STABILOMETER TEST DATA

During a quick stabilometer test, a relatively impermeable and saturated soil undergoes deformation without appreciable volume change. If, however, the soil contains air, loading tends to compress the air, according to Boyle's law, with consequent reductions of volume and height of the sample.

The known decrease in height due either to compression of air or its escape is not considered as deformation in computing the modulus of deformation, C . In computing this modulus, it is assumed that the soil deforms at constant volume in which case μ , Poisson's ratio, is $\frac{1}{2}$.

Reference is made to figure 2, which illustrates three distinctly different types of soil behavior during the

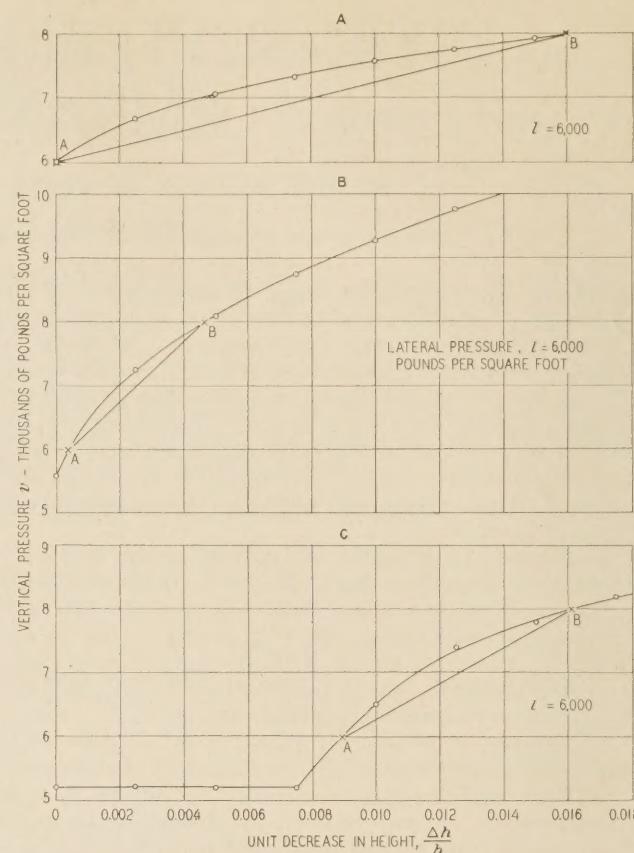


FIGURE 2.—STRESS-STRAIN CURVES FROM TRIAXIAL COMPRESSION TESTS.

early period of loading in the stabilometer or triaxial compression device. In figure 2-A, with a lateral pressure of 6,000 pounds per square foot, the developed vertical pressure with the machine in fixed position was also 6,000 pounds per square foot. The point, $v=l=6,000$ therefore falls on the axis of v , vertical pressure, which is the axis of zero $\frac{\Delta h}{h}$. Figure 2-B illustrates another type of behavior. Here the developed v with the machine in fixed position was 5,600 pounds per square foot with l , the lateral pressure, equal to 6,000. The point A for $v=l=6,000$ is not on the $\frac{\Delta h}{h}=0$ axis but corresponds to a value of 0.0004 for $\frac{\Delta h}{h}$. Figure 2-C illustrates still another type of behavior. Here the developed v with the machine in fixed position was 5,200 for $l=6,000$ and under this system of initial stresses, the height h decreased until the value of $\frac{\Delta h}{h}$ became 0.0075. Then as load was applied and v was increased, the sample began to deform. At the point A for $v=l=6,000$, $\frac{\Delta h}{h}$ is seen to be 0.0089.

Figure 2-A is characteristic of a relatively soft saturated soil of low permeability; figure 2-B is characteristic of a relatively stiff saturated soil of low permeability; and figure 2-C is characteristic of a stiff soil that contains an appreciable volume of air.

The horizontal distance from the point A, where $v=l$, to the $\frac{\Delta h}{h}=0$ axis is zero for curve A in figure 2, 0.0004

for curve B, and 0.0089 for curve C. These variable distances denote reductions in h due to volume changes without distortion and therefore represent small consolidations of the sample that are usually unavoidable, appear in their entirety, and are completely accounted for in consolidation tests. Thus the horizontal distances from A to the $\frac{\Delta h}{h}=0$ axis properly fall in the category of S_c settlement. The change in $\frac{\Delta h}{h}$ beyond the point A is indicative of distortion without volume change (S_L settlement).

The modulus of deformation, C , is taken as a secant modulus in this paper and is the slope of the secant line drawn from the initial point A, figure 2, where $l=v$, to another point B, determinable from the conditions of the particular problem. Since there are an infinite number of points beyond A, there may be an infinite number of secant lines and of moduli C . There is but one secant line, however, for a specific problem fixing the point B. Thus the nature of the problem may be such that the value of $v-l$ at the point B is 2,000 pounds per square foot. This is illustrated in figure 2. If $l=6,000$ and $v-l=2,000$, then $v=8,000$, the ordinate value of the point B, figure 3. In figure 2-A, the slope of AB is $\frac{8,000-6,000}{0.0161-0} = 125,000$ pounds per square foot = C . In figure 2-B, the slope of AB is $\frac{8,000-6,000}{0.0047-0.0004} = 465,000$ pounds per square foot = C .

In figure 2-C, the slope of AB is $\frac{8,000-6,000}{0.0161-0.0089} = 278,000$ pounds per square foot = C , rounding the value of C to the nearest 1,000 pounds per square foot.

For any vertical load greater than that corresponding to the initial point, A, figure 2, it is necessary to correct for the changed horizontal cross-sectional area of the deformed sample in computing v . This correction is contained in the following expression for v :

$$v = \frac{P}{A} \left(1 - \frac{\Delta h}{h}\right) + l \quad (1)$$

where P is the net load, equal to the load on the plunger plus the weight of the plunger minus the product of the lateral pressure, l , and the cross-sectional area, a , of the plunger stem. A is the initial cross-sectional area of the sample.

SETTLEMENT OF FILL UNDER ITS OWN WEIGHT CONSIDERED

Figure 3 illustrates a symmetrical earth fill with equal slopes. The Y direction is the direction of the length of the fill which is perpendicular to the plane of the figure. If the fill is long in comparison with its width, displacement in the Y direction is zero. The Z direction is the vertical one and OZ (fig. 3) is the axis of symmetry of any vertical cross section that is assumed to be of unit thickness in the Y direction. In the plane of the diagram (fig. 3) the X direction is horizontal. The directions are indicated by the coordinates, X , Y , and Z . The origin, 0, is at the base of the fill and on the vertical axis of symmetry. The letters x , y , and z denote both direction and distance whereas X , Y , and Z denote direction only. The normal stresses p_x and p_z act in the X and Z directions, respectively, and their values at any point depend upon the coordinates x and z of the point. The shearing stress, s_{xz} , acts in the X direction and in a plane to which the Z direction

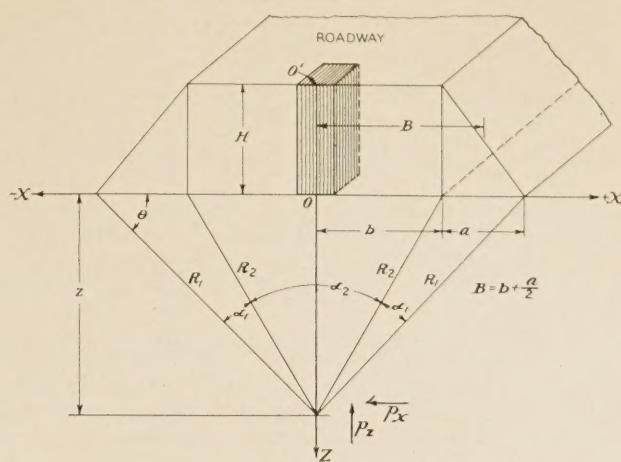


FIGURE 3.—DIAGRAM OF SYMMETRICAL EARTH FILL.

is perpendicular. The shearing stress, s_{xz} , acts in the Z direction and in a plane to which the X direction is perpendicular. For a condition of equilibrium, $s_{xz} = s_{zx}$.

The fill problem, figure 3, involves two dimensions, that is, only soil movements in the X and Z directions are of concern. The letter V designates the displacement of a particle at a point (x, z) in the Z direction and U designates the displacement of the particle at the same point in the X direction. The rate of change of V with respect to z , when x is constant, is $\frac{\partial V}{\partial z}$ and denotes the strain in the Z direction. The rate of change of U with respect to x , at constant z , is $\frac{\partial U}{\partial x}$, the strain in the X direction. The strain in the Y direction is zero. The problem is one of plane strain or deformation. From Hooke's law and Nadai's principle,

$$\frac{\partial V}{\partial z} = \frac{1}{C} [p_z - \mu(p_x + p_y)] \quad (2)$$

$$\text{strain in } Y \text{ direction} = 0 = \frac{1}{C} [p_y - \mu(p_x + p_z)] \quad (3)$$

$$\text{and} \quad \frac{\partial U}{\partial x} = \frac{1}{C} [p_x - \mu(p_y + p_z)] \quad (4)$$

Nadai refers to C as a "constant" rather than a modulus and, for a material that undergoes distortion without change in volume, takes μ as $1/2$.

Then, according to Nadai's principle, applicable to flow in a plastic mass, equation 2 would become

$$\frac{\partial V}{\partial z} = \frac{1}{C} [p_z - 1/2(p_x + p_y)] \quad (5)$$

Consider the column of earth, $00'$ on the vertical axis of symmetry in figure 3. For the present, consider the origin as moved from 0 to $0'$. Any vertical distance, z , is then considered as directed downward from the roadway which is the horizontal plane containing $0'$. Let w denote the weight per cubic foot of fill material assumed to be homogeneous. Then at any depth z in the column $0'0$, $p_z = wz$ and $p_x = K'wz$ where K' is the ratio of lateral to vertical pressure at the depth z . From equation 3

$$p_y = \mu(p_x + p_z) = \mu(K'wz + wz)$$

Substituting this value for p_y in equation 2

$$\frac{\partial V}{\partial z} = \frac{1}{C} [wz - \mu(K'wz + \mu K'wz + \mu wz)]$$

$$\text{or } \frac{\partial V}{\partial z} = \frac{wz}{C} \left[1 - \mu K' - \mu^2 (K' + 1) \right] \quad (6)$$

Integrating equation 6, assuming K' constant,

$$V = \frac{wz^2}{2C} \left[1 - \mu K' - \mu^2 (K' + 1) \right] + f(x) \quad (7)$$

On the axis of symmetry, $f(x)$, a function of x alone, becomes a constant, K_1 , and for points on this axis,

$$V = \frac{wz^2}{2C} \left[1 - \mu K' - \mu^2 (K' + 1) \right] + K_1 \quad (8)$$

If it is assumed that $V=0$ at $z=H$, a condition which will exist if the undersoil is unyielding and there is only the settlement S_L of the fill to consider, K_1 may then be evaluated so that equation 8 becomes

$$V = \frac{w}{2C} (z^2 - H^2) [1 - \mu K' - \mu^2 (K' + 1)] \quad (9)$$

and for $\mu=1/2$,

$$V = \frac{3w}{8C} (1 - K') (z^2 - H^2) \quad (10)$$

Here V denotes the downward displacement of a soil particle on the axis of symmetry and at any depth z from the roadway. The greatest vertical displacement is at $z=0$ at 0'. At this point,

$$V = S_L = -\frac{3w}{8C} H^2 (1 - K') \quad (11)$$

The use of equation 11 is limited by the fact that there is no sure way of determining K' . Theoretically, its value is greater than 0 and less than 1. If $K'=0$ is taken, then S_L is a maximum value and on the side of safety.

SETTLEMENT S_L OF THE UNDERSOIL DETERMINED

The origin is now taken at the point 0, figure 3. The angles, α_1 and α_2 of figure 3 are expressed in radians and from the diagram it is evident that

$$2\alpha_1 + \alpha_2 = 2 \operatorname{arc cot} \frac{z}{a+b}$$

and

$$\alpha_1 = \operatorname{arc cot} \frac{z}{a+b} - \operatorname{arc cot} \frac{z}{b}$$

It has been shown² that on the axis of symmetry, OZ,

$$p_z = \frac{p}{\pi} \left(2\alpha_1 + \alpha_2 + \frac{2b}{a} \alpha_1 \right) \quad (12)$$

and

$$p_x = \frac{p}{\pi} \left(2\alpha_1 + \alpha_2 + \frac{2b}{a} \alpha_1 - \frac{4z}{a} \log_e \frac{R_1}{R_2} \right) \quad (13)$$

where p_x and p_z are normal stresses (at any depth z on OZ) due solely to the fill load.

From equation 3, $p_y = \mu(p_x + p_z)$ and substituting this value for p_y in equation 2,

$$\frac{\partial V}{\partial z} = \frac{1}{C} [p_z - \mu(p_x + \mu p_x + \mu p_z)] \quad (14)$$

By substituting equations 12 and 13 in 14 one obtains

² See footnote 2, p. 161.

$$\frac{\partial V}{\partial z} = \frac{p}{\pi C} \left[(1 - \mu - 2\mu^2) \left(2 \operatorname{arc cot} \frac{z}{a+b} + \frac{2b}{a} \operatorname{arc cot} \frac{z}{a+b} \right. \right. \\ \left. \left. - \frac{2b}{a} \operatorname{arc cot} \frac{z}{b} \right) + \mu(1+\mu) \frac{4z}{a} \log_e \frac{\sqrt{(a+b)^2 + z^2}}{\sqrt{b^2 + z^2}} \right] \quad (15)$$

It must be remembered that on integrating equation 15, it is desired to know V on the axis of symmetry and hence, $x=0$. The last term of equation 15 is the only one giving difficulty in integration. This may be integrated by parts and by the transformation,

$$z = (a+b) \tan \theta$$

where θ is the angle shown in figure 3.

The vertical displacement, V , of a soil particle at the point 0, figure 3, and S_L , are identical in magnitude. In general, however, V refers to the vertical displacement of a particle at any point and not just at 0. Hence, it is a special value of V , namely its value at 0, the sum of all the strains from 0 to a given depth z , that is equal to S_L .

By, integration of equation 15 between the limits, 0 and z , one obtains

$$V \text{ at } 0 = \frac{p}{\pi C} \left\{ (1 - \mu - 2\mu^2) \left(2 + \frac{2b}{a} \right) \right. \\ \left[z \operatorname{arc cot} \frac{z}{a+b} + \frac{a+b}{2} \log_e \left(1 + \frac{z^2}{(a+b)^2} \right) \right] \\ - (1 - \mu - 2\mu^2) \left(\frac{2b}{a} \right) \left[z \operatorname{arc cot} \frac{z}{b} + \frac{b}{2} \log_e \left(1 + \frac{z^2}{b^2} \right) \right] \\ + \frac{4\mu(1+\mu)}{a} \left[\frac{z^2}{2} \log_e \frac{\sqrt{z^2 + (a+b)^2}}{\sqrt{z^2 + b^2}} \right. \\ \left. + \frac{(a+b)^2}{2} \log_e \frac{\sqrt{z^2 + (a+b)^2}}{a+b} - \frac{b^2}{2} \log_e \frac{\sqrt{z^2 + b^2}}{b} \right] \right\} \quad (16)$$

S_L denotes the diminution in thickness of a given depth of undersoil due to lateral displacement, and

for $\mu=1/2$ equation 16 reduces to

$$S_L = V \text{ at } 0 = \frac{3p}{\pi Ca} \left[\frac{z^2}{2} \log_e \frac{\sqrt{z^2 + (a+b)^2}}{\sqrt{z^2 + b^2}} \right. \\ \left. + \frac{(a+b)^2}{2} \log_e \frac{\sqrt{z^2 + (a+b)^2}}{a+b} - \frac{b^2}{2} \log_e \frac{\sqrt{z^2 + b^2}}{b} \right] \quad (17)$$

It is convenient to obtain solutions of equations 12 and 16 by graphical methods. With reference to figure 3, let $B=b+\frac{a}{2}$. Then for $a=0$, the trapezoidal load becomes a uniform strip load and $\frac{b}{B}=1$. For $b=0$,

the load diagram becomes triangular and $\frac{b}{B}=0$. All of the possible symmetrical trapezoidal load diagrams are then contained within the limiting cases, $\frac{b}{B}=0$ and $\frac{b}{B}=1$. Now write for equation 12,

$$p_z = pf$$

where $f = \frac{1}{\pi} \left(2\alpha_1 + \alpha_2 + \frac{2b}{a} \alpha_1 \right)$.

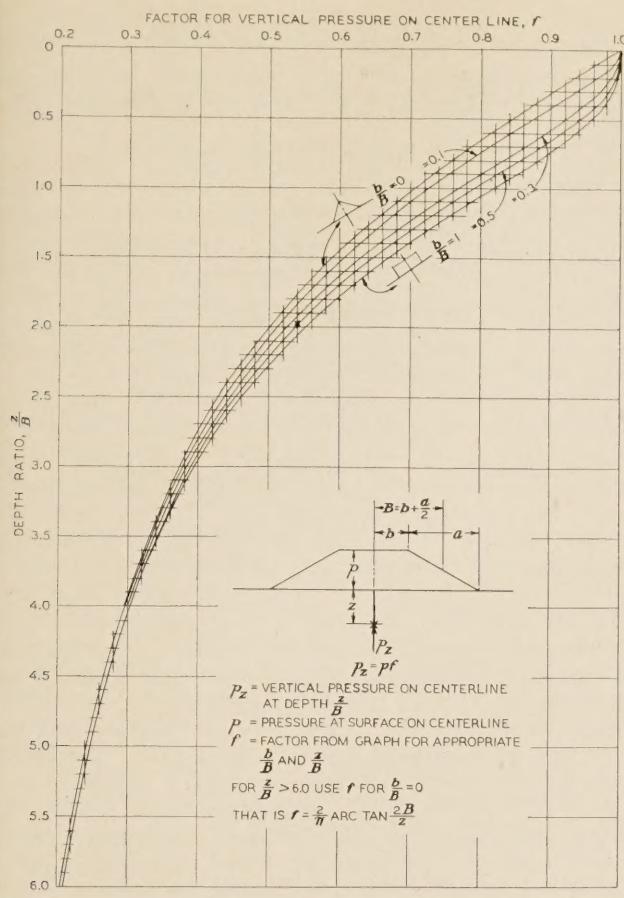


FIGURE 4.—GRAPH OF VERTICAL PRESSURE ON CENTERLINE UNDER A SYMMETRICAL FILL.

The value of f depends on the depth z of any point on the centerline and on the distances, a and b (fig. 3). The curves in figure 4 are obtained by plotting values

of f against the "depth ratio," $\frac{z}{B}$. The five curves of this figure are for the ratios of $\frac{b}{B}$ equal to 0, 0.1, 0.3, 0.5, and 1.0. For ratios that are intermediate, a value for f corresponding to a given $\frac{z}{B}$ ratio may be obtained by interpolation. For depths such that $\frac{z}{B}$ exceeds 6.0, the value for f is taken from the formula for $\frac{b}{B} = 0$, regardless of what the actual value of $\frac{b}{B}$ may be since there is practically complete coincidence of all the curves at $\frac{z}{B} = 6.0$.

In fills having the same B and the same height, H , the vertical cross-sectional area is the same. Hence, it is convenient to use the value B as a basis for constructing charts such as figure 4 for use in computations.

Equation 16 may be written

$$S_L = \frac{p}{C} \left(b + \frac{a}{2} \right) F = \frac{pB}{C} F \quad (18)$$

where

$$F = \frac{1+\mu}{\pi(1-b/B)} \left[(1-\mu) \left[(2-b/B)^2 \log_e \sqrt{1 + \frac{(z/B)^2}{(2-b/B)^2}} - (b/B)^2 \log_e \sqrt{1 + \frac{(z/B)^2}{(b/B)^2}} \right] + \mu(z/B)^2 \log_e \sqrt{\frac{(z/B)^2 + (2-b/B)^2}{(z/B)^2 + (b/B)^2}} + (1-2\mu)z/B \left[(2-b/B) \text{arc cot } \frac{z/B}{2-b/B} - (b/B) \text{arc cot } \frac{z/B}{b/B} \right] \right]$$

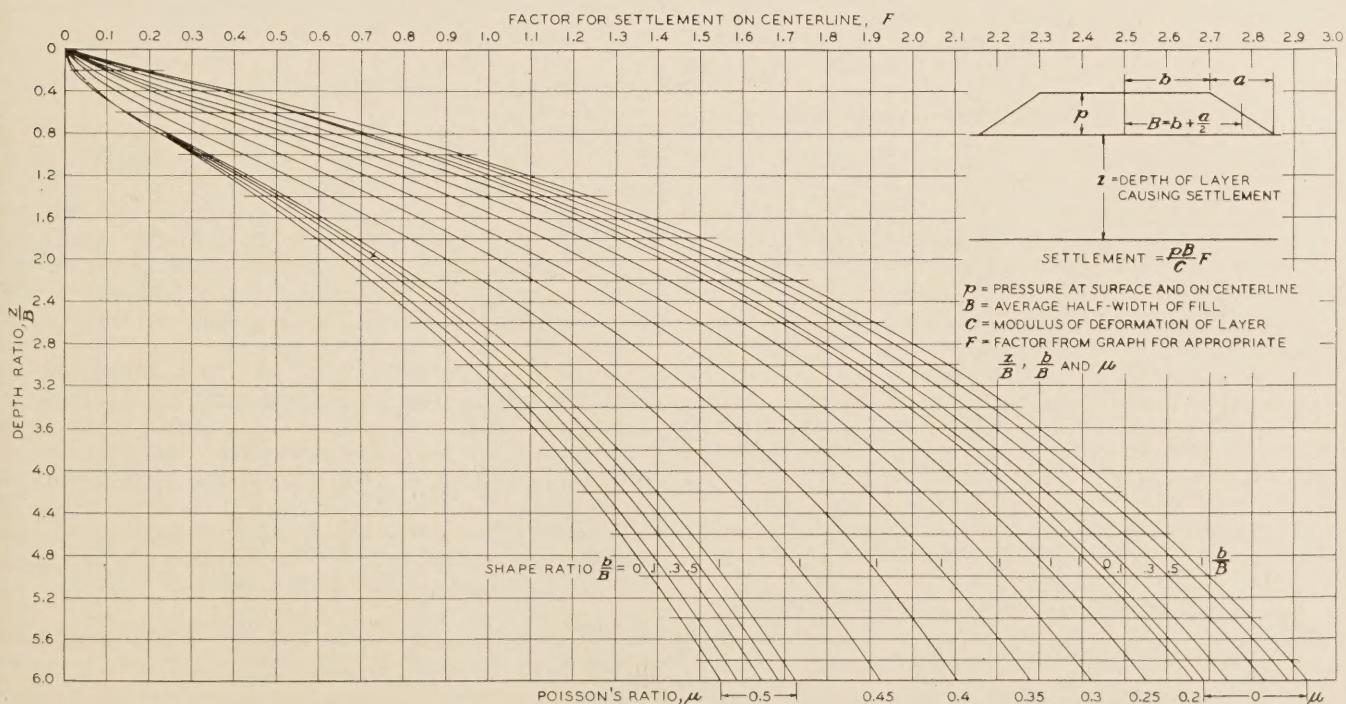


FIGURE 5.—GRAPH OF SETTLEMENT ON CENTERLINE UNDER A SYMMETRICAL FILL.

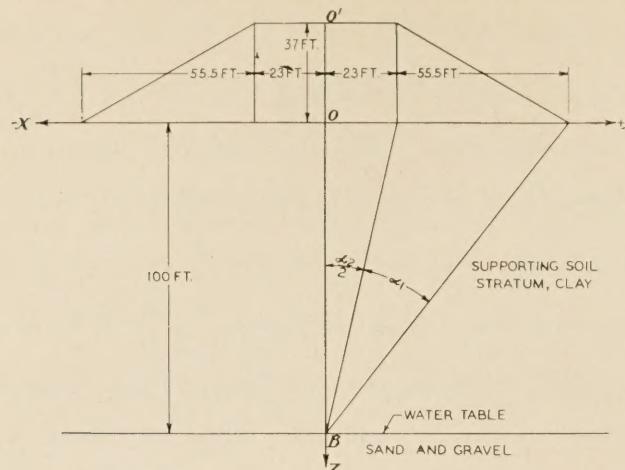


FIGURE 6.—SYMMETRICAL FILL SUPPORTED BY CLAY.

The value of F in equation 18 depends on μ , Poisson's ratio. The curves of figure 5 were constructed for values of μ varying from 0 to $\frac{1}{2}$ by plotting values of $\frac{z}{B}$ against F for different values of $\frac{b}{B}$. A numerical example will be given illustrating the use of the chart, figure 5.

COMPUTATION OF S_L ILLUSTRATED

Consider the fill, figure 6, of height, $H=37$ feet, with a $1\frac{1}{2}$ to 1 slope and supported by a clay stratum 100 feet thick which is underlaid by sand and gravel. The supporting soil and fill material have different properties. The average weight per cubic foot of the compacted fill material is $w=126$ pounds. The width of the roadway is 46 feet so that b (fig. 3) is 23 feet. The distance, a , is $1\frac{1}{2} \times 37 = 55.5$ feet. The value of $B=b+\frac{a}{2}=$

$23 + \frac{55.5}{2} = 50.8$ feet and the ratio, $\frac{b}{B}$, is $\frac{23}{50.8} = 0.453$. The value of p is $wH=37 \times 126 = 4,662$ pounds per square foot. The thickness of the deformable clay layer is 100 feet and the depth ratio, $\frac{z}{B}$, is $\frac{100}{50.8} = 1.97$.

In previous publications^{2,3} it has been shown that the greatest shearing stress anywhere in the undersoil below a symmetrical fill is $0.32 p$. This greatest shearing stress is located at a distance equal approximately to B below the point 0, figure 3, and is equal to $\frac{1}{2}(v-l)$ where v is the vertical pressure and l the lateral pressure at the point of greatest shearing stress.

Then since $\frac{1}{2}(v-l)=0.32 p$, $v-l=0.64 p$ and theoretically this is the greatest $v-l$ value that exists anywhere in the supporting earth below the embankment. For $p=4,662$ pounds per square foot, the greatest value of $v-l$ is $0.64 \times 4,662 = 2,984$ pounds per square foot. With reference now to figure 7 which shows the v versus $\frac{\Delta h}{h}$ characteristics of the undersoil for $l=0$ and $l=4,180$ pounds per square foot, the point B is determined for each curve. For $v-l=2,984$, when $l=0$, $v=2,984$ and for $l=4,180$, $v=2,984+4,180=7,164$ pounds per square foot. The ordinate values of B are then 2,984 for $l=0$ and 7,164 for $l=4,180$. The value of $\frac{\Delta h}{h}$ at the initial point A for $l=0$ is -0.0001. The initial point A for $l=4,180$ is at that point of the upper curve where $v=l=4,180$. For $l=4,180$, the slope of AB , upper curve of figure 2, is $\frac{7,164-4,180}{0.0310-0.0036}=109,000$ pounds per square foot = C . For $l=0$, the slope of AB ,

^{2,3} See footnotes 2 and 3, p. 161.

(Continued on page 172)

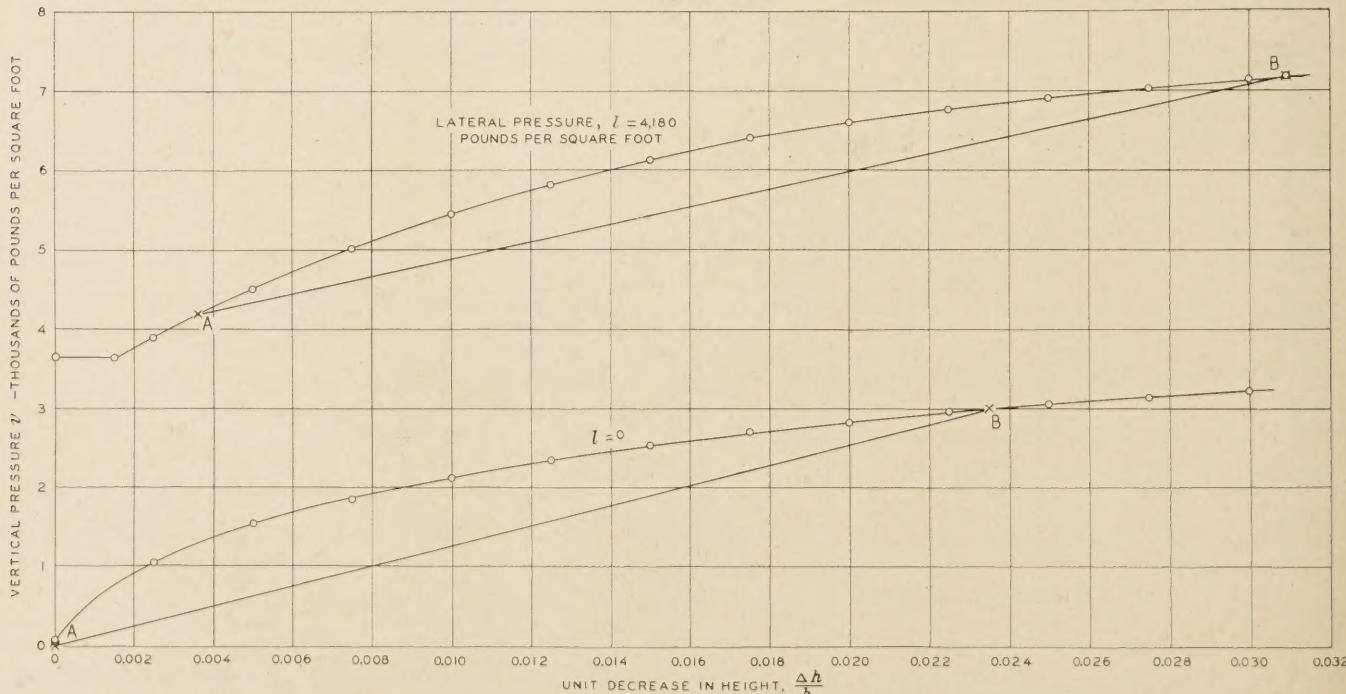


FIGURE 7.—TRIAXIAL TEST DATA FOR FOUNDATION SOIL.

HORIZONTAL FORCES AND MOMENTS ON BRIDGE ABUTMENTS AND RETAINING WALLS

EFFECT OF CONCENTRATED MOVING LIVE LOADS ON THE BACKFILL

Reported by E. I. FIESENHEISER, Assistant Bridge Engineer, District 4, Public Roads Administration

THE PURPOSE of this paper is to present methods of computing the overturning effect of live loads by taking into account the distribution of stresses through the soil. The paper presents:

- (1) Exact equations for the force and moment based on the theoretical variation of stress.
- (2) Simplified approximate equations sufficiently accurate for most cases.
- (3) A determination of the position of the live load for maximum moment.
- (4) Charts for use in computing.

In designing bridge abutments and retaining walls the horizontal pressure of the backfill material on the wall must be taken into account. The amount of this pressure and its overturning moment are usually determined by assuming that the pressure increases uniformly with the depth of fill. This assumption results in either a triangular or trapezoidal pressure diagram.

In dealing with the problem of truck wheels or concentrated live loads on top of the backfill or roadway it is customary to assume an added depth of fill or surcharge. The loads are assumed to be distributed over an area and the depth of surcharge is taken as the depth of a volume of backfill material equal to the loads in weight.

The moment effect of the live loads upon the wall has been a matter for conjecture and the method of adding a surcharge has doubtless been used for lack of a better or more precise method. However, recent experiment and progress in the field of soil mechanics have pointed the way to a very different solution of this problem.

STRESS DISTRIBUTION EQUATIONS DEVELOPED

When taking into account the stress distribution in the soil reference is usually made to the work of Joseph Boussinesq, a noted elastician, who derived equations for the stresses in the interior of an elastic solid. The conditions assumed by Boussinesq are a semi-infinite, elastic, isotropic solid bounded by a plane surface upon which a single concentrated load acts in a direction perpendicular to the plane surface. Published equations,¹ give the normal and shearing components of stress in the interior of the solid. However, these stress components are only those acting on planes parallel to the surface plane.

Assuming the horizontal surface of the backfill to be the boundary plane and keeping in mind that the plane of the wall or abutment is vertical, an expression is needed for the horizontal component of stress acting upon a plane perpendicular to the surface plane. Such an expression in terms of rectangular coordinates with the origin at the load point is $\sigma_x = \frac{3P}{2\pi} \frac{x^2 z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$, for the stress in the x -direction, the z -axis being vertical and the x - and y -axes horizontal. This is a simplified

expression derived by assuming the term Poisson's ratio equal to one-half, which is in accordance with a fundamental principle of soil mechanics that the soil particles are incompressible or incapable of a change in volume. Equations from which it is derived may be found in texts on the theory of elasticity.²

Actual working conditions for which the above expression would be applicable would be a soil of perfect elasticity with elastic properties the same in all directions (conditions of an isotropic soil). Some soils may very nearly approach these conditions, while others may be far from elastic and isotropic. However, with regard to retaining walls, experiments have indicated that the shape of the pressure variation curve is similar to that obtained from the theoretical expression and that the quantitative differences may be taken into account by the use of empirical constants dependent upon the type of soil.

In experiments on retaining walls conducted at the Iowa Engineering Experiment Station, Ames, Iowa, by M. G. Spangler³ the horizontal pressure due to concentrated surface loads was measured. By transforming the theoretical expression an empirical equation was devised which gives pressure intensities corresponding to the measured intensities. The equation devised is

$$h_c = \frac{kP}{x^n (x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

in which k is a constant depending upon the type of soil and n is a constant depending upon the relative rigidity of the wall and backfill. Obviously more experimenting is needed with different types of soil before values of these constants can be determined for all cases, since these experiments were made with gravel only.

HORIZONTAL FORCES AND OVERTURNING MOMENTS COMPUTED

In applying the equation to the computation of horizontal forces and overturning moments working values of k equal to 1.3 and n equal to 1/4 will be used, assuming a gravel backfill and conditions similar to those in the experiments conducted by Spangler. If values for k and n can be determined for other materials and conditions, such values may be substituted in the equations to be derived. Forces and moments will be directly proportional to the constant k and inversely to x^n , x being the distance from the load to the wall. (See fig. 1.)

As shown in figure 1, with the origin directly under the load, the coordinates of a point on the wall are x , y , and z , where

x =horizontal distance from the load to the wall.

y =horizontal distance from the load parallel to the wall.

z =vertical distance below the top of the fill.

$R = \sqrt{x^2 + y^2 + z^2}$.

P =the concentrated load.

¹ Application des Potentiels a L'étude de L'équilibre et du Mouvement des Solides Elastique, by M. J. Boussinesq. Gauthier-Villars. Paris. 1885. p. 104.

² For the solution in terms of cylindrical coordinates see Theory of Elasticity, by S. Timoshenko. McGraw Hill, New York. 1934. pp. 328-331.

³ Horizontal Pressures on Retaining Walls Due to Concentrated Surface Loads,

b M. G. Spangler. Iowa Engineering Experiment Station Bulletin 140. 1938.

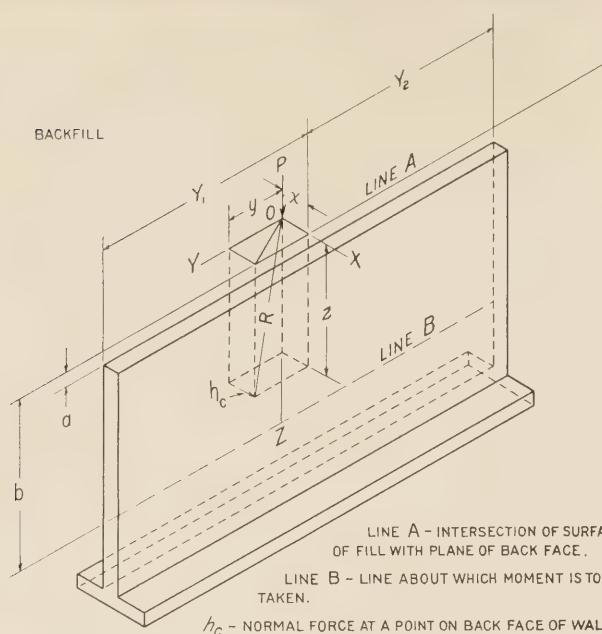


FIGURE 1.—DIAGRAM SHOWING RETAINING WALL AND LOAD ON BACKFILL.

a = vertical distance from top of fill to top of wall.
 b = vertical distance from top of fill to a horizontal line where the moment is desired.

M = moment at the line b distance below top of fill.
 H = total horizontal force acting on the wall above the line.
 h_c = intensity of horizontal pressure at any point on the wall.

To get a clear picture of the stress variation expressed by the equation, curves may be plotted holding x and y constant while z varies (see fig. 2) and holding x and z constant while y varies (see fig. 3).

In dealing with a bridge abutment attention is called to the fact that the top or surface of the backfill is not always at the top of the wall. A part of the earth pressure may be taken by the floor slab or diaphragms at the end of the bridge and transferred as a thrust through the superstructure. In such a case only the horizontal force acting on the area of the wall unit is of interest. Accordingly a distance a from the surface of the fill to the top of the wall is introduced to take care of this condition.

$$\text{Referring to figure 2, } h_c = \frac{kP}{x^n} \cdot \frac{x^2 z}{(x^2 + y^2 + z^2)^{\frac{n}{2}}}$$

The differential area of wall $dA = dy dz$ and the force $dH = h_c dA = \frac{kP}{x^n} \cdot \frac{x^2 z dy dz}{(x^2 + y^2 + z^2)^{\frac{n}{2}}}$

Taking as z -limits the distances b and a and as y -limits the distances Y_1 and Y_2 ,

$$H = kPx^{(2-n)} \int_a^b \int_{Y_2}^{Y_1} \frac{z dy dz}{(x^2 + y^2 + z^2)^{\frac{n}{2}}} \quad (1)$$

Integrating in z -direction and substituting limits,

$$H = \frac{kPx^{(2-n)}}{3} \left\{ \int_{Y_2}^{Y_1} (a^2 + x^2 + y^2)^{-\frac{n}{2}} dy - \int_{Y_2}^{Y_1} (b^2 + x^2 + y^2)^{-\frac{n}{2}} dy \right\}$$

Integrating this expression in the y -direction and substituting limits,

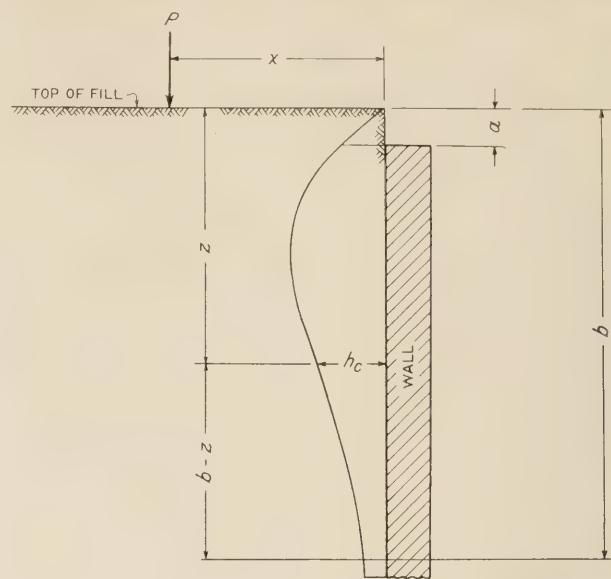


FIGURE 2.—STRESS VARIATION, x AND y CONSTANT.

$$H = \frac{kPx^{(2-n)}}{3}$$

$$\left\{ Y_1 \left[\frac{1}{(a^2 + x^2) \sqrt{a^2 + x^2 + Y_1^2}} - \frac{1}{(b^2 + x^2) \sqrt{b^2 + x^2 + Y_1^2}} \right] - Y_2 \left[\frac{1}{(a^2 + x^2) \sqrt{a^2 + x^2 + Y_2^2}} - \frac{1}{(b^2 + x^2) \sqrt{b^2 + x^2 + Y_2^2}} \right] \right\} \quad (2)$$

In deriving an equation for the overturning moment the differential of moment about a line b distance below the top of fill is

$$dM = \frac{kP}{x^n} \frac{x^2 z (b-z) dy dz}{(x^2 + y^2 + z^2)^{\frac{n+1}{2}}}$$

and the total moment

$$M = \frac{kPx^2}{x^n} \int_a^b \int_{Y_2}^{Y_1} \frac{z(b-z) dy dz}{(x^2 + y^2 + z^2)^{\frac{n+1}{2}}} \quad (3)$$

Integrating first in the z -direction and substituting the limits b and a ,

$$M = \frac{kPx^{(2-n)}}{3} \int_{Y_2}^{Y_1} \frac{1}{(x^2 + y^2)} \left[\frac{a^3 + bx^2 + by^2}{(a^2 + x^2 + y^2)^{\frac{n+1}{2}}} - \frac{b}{(b^2 + x^2 + y^2)^{\frac{n+1}{2}}} \right] dy$$

Integrating now in the y -direction, substituting the limits Y_1 and Y_2 , and simplifying, the total moment becomes

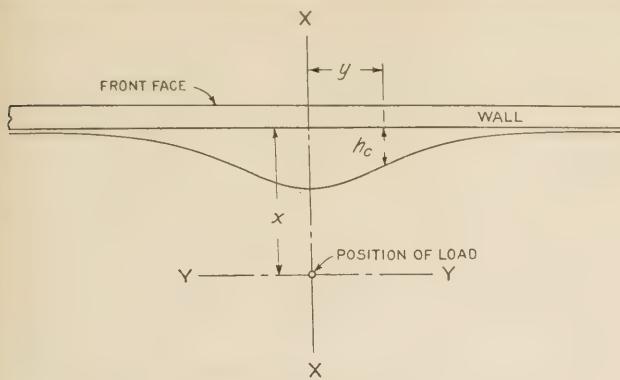
$$M = \frac{kPx^{(2-n)}}{3} [(a^3 + bx^2)A + b(C-B)] \quad (4)$$

in which

$$A = \frac{1}{a^2} \left(\frac{\alpha}{ax} - \frac{\gamma}{a^2 + x^2} \right) \quad (5)$$

$$B = \frac{\phi}{bx} \quad (6)$$

$$C = \frac{1}{a^2} \left(\gamma - \frac{x\alpha}{a} \right) \quad (7)$$

FIGURE 3.—STRESS VARIATION, x AND z CONSTANT.

and

$$\gamma = \frac{Y_1}{\sqrt{a^2 + x^2 + Y_1^2}} - \frac{Y_2}{\sqrt{a^2 + x^2 + Y_2^2}} \quad (8)$$

$$\alpha = \tan^{-1} \left[\frac{a}{x} \frac{Y_1}{\sqrt{a^2 + x^2 + Y_1^2}} \right] - \tan^{-1} \left[\frac{a}{x} \frac{Y_2}{\sqrt{a^2 + x^2 + Y_2^2}} \right] \quad (9)$$

$$\phi = \tan^{-1} \left[\frac{b}{x} \frac{Y_1}{\sqrt{b^2 + x^2 + Y_1^2}} \right] - \tan^{-1} \left[\frac{b}{x} \frac{Y_2}{\sqrt{b^2 + x^2 + Y_2^2}} \right] \quad (10)$$

Referring to equations 8, 9, and 10 it should be noted that Y_1 and Y_2 are distances measured parallel to the wall from the load to the ends of the wall. When the load is between the ends, which will ordinarily be the case, Y_1 will be assumed positive and Y_2 negative. The result will be the addition of the two terms in obtaining the value of the angles α and ϕ and the term γ . Angles α and ϕ are expressed in radians. When computing these angles if a conversion table is not at hand the angles may first be found in degrees and decimals of a degree from tables of natural functions, then multiplied by the constant 0.017453 to change degrees to radians.

For the particular case when the top of the fill coincides with the top of the wall the term a becomes zero. To obtain the moment the original expression, equation 3, may be integrated direct between limits b and 0. An alternate method is to substitute zero for a in the final equations 4 to 10. In the latter case the result will be zero divided by zero for the terms A and C . These indeterminate forms may be evaluated in the usual manner by differentiating. Taking third derivatives of numerators and denominators the values of the fractions can be found. By either method the resulting equation will be:

For $a=0$,

$$M = \frac{kPx^{(2-n)}}{3} \cdot b(D-B) \quad (11)$$

in which the term

$$D = \frac{Y_1}{x^2 \sqrt{x^2 + Y_1^2}} - \frac{Y_2}{x^2 \sqrt{x^2 + Y_2^2}} \quad (12)$$

EQUATIONS APPLIED TO BRIDGE ABUTMENT PROBLEM

Equations 2 and 4 are expressions for the force and its overturning moment caused by a single load P concentrated at a point and placed at x distance from the wall. If the familiar principle of superposition is applied when there are more loads than one to deal with, the

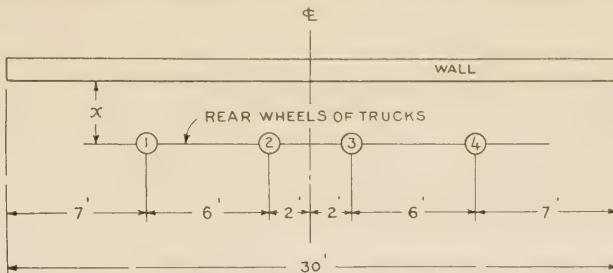


FIGURE 4.—PLAN OF ABUTMENT WALL, SHOWING POSITION OF REAR WHEELS OF 2 TRUCKS.

effects of the loads may be added together and the total moment will be the sum of the moments produced by the individual loads.

In applying the above equations to a bridge abutment a structure carrying two 10-foot traffic lanes is assumed. An abutment wall at least 30 feet long will ordinarily be required to support the two-lane superstructure. Such an arrangement of rear truck wheels as that indicated in figure 4 may be assumed.

Using the 30-foot wall as a basis for computing, a large number of solutions were worked out for the moment using different values for b and a . These computations revealed the fact that for relatively small values of a or x there is not much difference in the effect produced by wheels 1 and 4 and that produced by wheels 2 and 3. For example this difference was found to be less than 1 percent for the values $b=30$ feet and $a=0.5$ foot. In dealing with a wall of this length or greater it will usually be sufficiently accurate to compute the moment for a single load and multiply the result by the number of loads to obtain the total effect.

It can be seen that a single computation of force and moment by the above equations involves considerable work. Fortunately it is possible to derive much simpler equations which will be sufficiently accurate for the average wall. In a problem of this kind unnecessary refinements in calculation cannot be justified, especially if values of the soil constants cannot be determined with great exactness. The accuracy of the final result will depend upon experimental values of k and n . If infinite limits are assumed for y , letting $Y_1=+\infty$ and $Y_2=-\infty$, the following expressions are obtained:

$$H = \frac{2kPx^{(2-n)}}{3} \left[\frac{1}{a^2 + x^2} - \frac{1}{b^2 + x^2} \right] \quad (13)$$

$$M = \frac{2kPx^{(1-n)}}{3} \left[\frac{(b-a)x}{a^2 + x^2} + \tan^{-1} \frac{a}{x} - \tan^{-1} \frac{b}{x} \right] \quad (14)$$

For $a=0$, equation (14) becomes

$$M = \frac{2kPx^{(1-n)}}{3} \left[\frac{b}{x} - \tan^{-1} \frac{b}{x} \right] \quad (15)$$

It will be found that for a problem such as that of figure 4 equations 13, 14, and 15 will give results only slightly greater than those obtained from the exact equations 2 and 4 to 10, particularly for relatively small values of a and x . For example, for the values $b=30$ feet, $a=0.5$ foot, $x=1.18$ feet, the moment computed by equation 14 was found to be less than 1 percent greater than that computed by equations 4 to 10. Moments and forces determined by the simpler

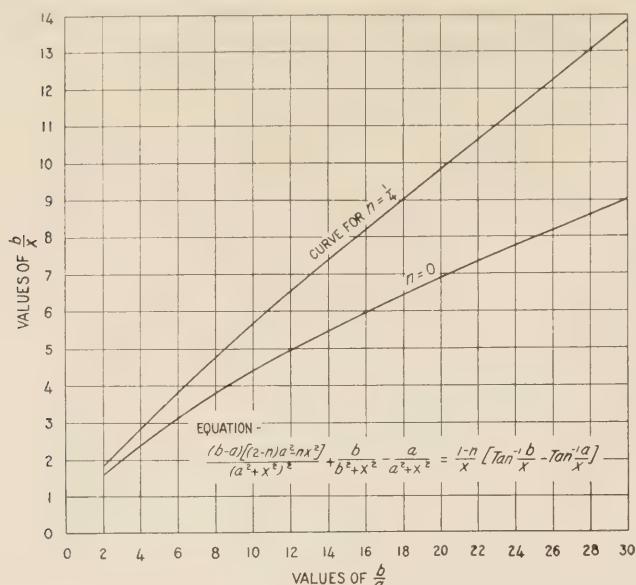


FIGURE 5.—CURVES SHOWING POSITION OF LOAD FOR MAXIMUM MOMENT, $a > 0$.

expressions will be on the safe side and it is believed that their use is justified.

Up to this point in the discussion the distance x has been treated only as a constant. However, for live loads moving toward or away from an abutment there is a value of x for which the moment is a maximum. To derive an equation for this value of x , equation 14 may be differentiated with respect to x and the result equated to zero. Performing these operations the following expression is obtained:

$$\frac{(b-a)[(2-n)a^2-nx^2]}{(a^2+x^2)^2} + \frac{b}{b^2+x^2} - \frac{a}{a^2+x^2} = \frac{1-n}{x} \left(\tan^{-1} \frac{b}{x} - \tan^{-1} \frac{a}{x} \right) \quad (16)$$

Substituting known values for b and a in the equation enables solving for x . Placing the load at this distance from the wall will then produce the maximum moment. If the value of n is taken as $\frac{1}{4}$, equation 16 becomes

$$\frac{(b-a)(7a^2-x^2)}{4(a^2+x^2)^2} + \frac{b}{b^2+x^2} - \frac{a}{a^2+x^2} = \frac{3}{4x} \left(\tan^{-1} \frac{b}{x} - \tan^{-1} \frac{a}{x} \right) \quad (17)$$

While solving for x appears to be involved, the equation may be handled practically by plotting two curves, one for each side of the equation. Extending the two curves until they intersect will give the solution, the point of intersection being the value of x for which the equality is true. The solution of equation 17 is given in figure 5 where the ratio $\frac{b}{x}$ is plotted against

the ratio $\frac{b}{a}$. When b and a are known, $\frac{b}{x}$ may be found from figure 5 and x may be determined from the ratio.

When the top of the fill coincides with the top of the wall the value of a is zero. For this particular case equation 16 for the maximum moment is of no use. It will be seen that for this case the moment approaches infinity as x approaches zero. This is due to the fact that in theory, as the point of application of the force

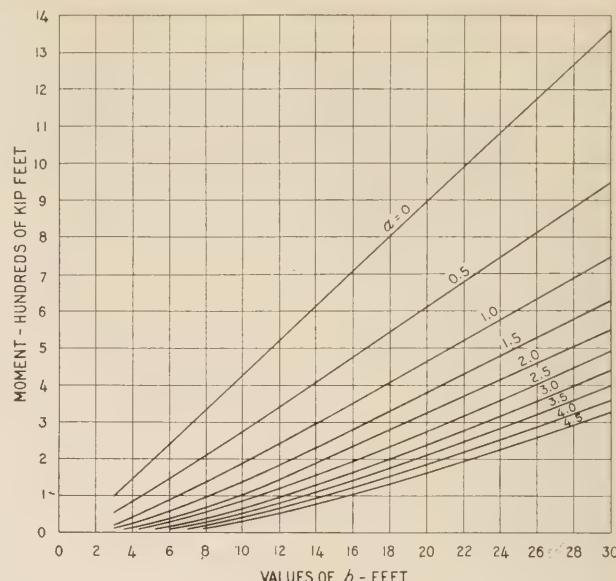


FIGURE 6.—MAXIMUM OVERTURNING MOMENTS AT VARIOUS DISTANCES FROM TOP OF FILL FROM THE REAR WHEELS OF TWO H-15 TRUCKS, $k=1.3$, $n=\frac{1}{4}$.

is approached, the finite force is acting upon an infinitely small area. Practically, however, the load P is not concentrated at a point but is distributed over an area. If the area of distribution is assumed to be a 15-inch circle, it is obvious that the value of x cannot be less than the radius of the circle. Accordingly, the minimum value of x was taken as $7\frac{1}{2}$ inches or 0.625 foot.

METHOD OF IMAGES MAY BE USED

Using four concentrated loads of 12,000 pounds each, a value of k equal to 1.3, n equal to $\frac{1}{4}$, and values of x for the maximum moment, the curves of figure 6 were plotted. It will be noted that the distance a has a considerable effect upon the moment, and that for large values of a the moment is greatly reduced.

In dealing with the standard H-loading it may be necessary in some cases to consider the smaller wheel concentrations 14 feet distant from the large wheels. It is unlikely, however, that two trucks headed in the same direction will enter a bridge at the same instant. Assuming that the four wheels shown in figure 4 are rear wheels of trucks, the front wheels of one truck will be on the fill and the front wheels of the other truck will be on the superstructure. Under such conditions the effect of the two front wheels on the fill will usually be very small in comparison with the effect of the four rear wheels which are close to the wall. However, if it is desired to include the effect of all wheels, the rear wheels may be placed at the point for maximum moment and with the trucks in this position a separate computation made for each set of loads. This position of the trucks should give moments sufficiently close to a maximum for design purposes.

When the total moment and the total horizontal force have been computed the height of the center of pressure may be determined by dividing the moment by the force. The ratio of this height to the height of wall, $(b-a)$, was found to vary from approximately 0.53 to 0.96 as the height of wall varies from 2 to 30 feet. Further, this ratio increases when the height of wall increases or when the distance a decreases. It is noted that the location of the center of pressure is not affected by either of the constants k or n .

Some designers may prefer to assume a soil of perfect elasticity and to use the equation based upon the theory of elasticity rather than the constants determined by experiment. In this case the method of images may be applied as explained by Dr. Raymond D. Mindlin.⁴ The constant n will drop out or become equal to zero and the constant k which appears in the above equations will become $\frac{3}{\pi}$. By this method the intensity of normal pressure at any point on the wall is simply double that given by the equation for the stress in an elastic, isotropic solid (see p. 167). The force and moment equations 2 to 15, and equation 16 for the maximum, may be used by making this change in the constants. The values obtained for the force and moment will be slightly less than for $k=1.3$, $n=1/4$, when the load is placed at the point for maximum moment.

Assuming a bridge abutment backfilled with gravel, the moment and horizontal force will be computed by the approximate equations. The conditions assumed will be a live load P of 24 kips, $b=20$ feet, $a=1$ foot, $k=1.3$, and $n=1/4$.

The moment, using equation 14, is

$$M = \frac{2}{3}kPx^{1-n} \left[\frac{(b-a)x}{a^2+x^2} - \left(\tan^{-1} \frac{b}{x} - \tan^{-1} \frac{a}{x} \right) \right]$$

From figure 5 for $\frac{b}{a}=20$, $\frac{b}{x}=9.82$.

Then $x=\frac{20}{9.82}=2.03$ feet, the position of the load for

maximum moment when $n=1/4$,

$$(2.03)^{1-n} = (2.03)^{3/4} = 1.70$$

and

$$\frac{2}{3}kPx^{1-n} = \frac{2}{3}(1.3)24(1.70) = 35.4$$

$$M = 35.4 \left[\frac{19(2.03)}{5.12} - (\tan^{-1} 9.82 - \tan^{-1} 0.492) \right].$$

$$\tan^{-1} 9.82 - \tan^{-1} 0.492 = (84^\circ 12') - (26^\circ 12') = 58.0^\circ = 0.01745(58.0) = 1.01 \text{ radians.}$$

$$M = 35.4(7.52 - 1.01) = 230 \text{ foot-kips.}$$

This moment may be determined from figure 6. The value read from the chart should be divided by 2 since a load of 48 kips was used in computing values for the curves.

By equation 13

$$H = \frac{2}{3}kPx^{1-n} \left[\frac{1}{a^2+x^2} - \frac{1}{b^2+x^2} \right],$$

the horizontal force

$$H = 35.4(2.03) \left[\frac{1}{5.12} - \frac{1}{404.1} \right] = 71.8(0.1952 - 0.0025) = 13.8 \text{ kips.}$$

If an elastic, isotropic backfill material is assumed and the live load effect on the wall is taken into account by using the method of images the only change in the problem will be in the assumed values of the constants. The new values will be $k=\frac{3}{\pi}$ and $n=0$. Again taking $P=24$ kips, $b=20$ ft., and $a=1$ foot, from figure 5 for $\frac{b}{a}=20$, $\frac{b}{x}=6.90$.

Then $x=\frac{20}{6.90}=2.80$ feet, the position of the load for a maximum moment when $n=0$.

$$\frac{2}{3}kPx^{1-n} = \frac{2}{3} \cdot \frac{3}{\pi} (24) 2.80 = 42.8$$

$$M = 42.8 \left[\frac{19(2.80)}{8.84} - (\tan^{-1} 6.90 - \tan^{-1} 0.357) \right]$$

$$\begin{aligned} \tan^{-1} 6.90 - \tan^{-1} 0.357 &= (81^\circ 46') - (19^\circ 38') \\ &= 62.13^\circ = 0.01745(62.13) = 1.08 \text{ radians} \end{aligned}$$

$$M = 42.8(6.02 - 1.08) = 211 \text{ foot-kips}$$

and

$$\begin{aligned} H &= 42.8(2.80) \left[\frac{1}{8.84} - \frac{1}{407.8} \right] \\ &= 119.5 (0.1130 - 0.0024) = 13.2 \text{ kips.} \end{aligned}$$

The above computations were made with an ordinary 10-inch slide rule using C, D, L, and T scales.

CONCLUSIONS

In the preparation of figure 6 the value of k was taken as 1.3 as this is the value suggested as safe for gravel material similar to that used in the experiments.³ In the absence of further experimental data this value might be used for the average gravel backfill.

Regarding the experimental constant n , the value of $1/4$ was found to agree most closely with the experimental data reported by Spangler. Since this constant is written as an exponent of x , its value affects the position of the load for maximum moment. With reference to the curves of figure 5 it will be noted that for a maximum moment the load must be placed farther from the wall for n equal to zero than for n equal to $1/4$.

The reader may question when to use the method involving the empirical constants and when to use the method of images. It is the writer's opinion that when dealing with a backfill whose soil properties are unknown it would be more logical to assume elasticity and use the method of images than to make a guess at a value for k which had not been verified by experiment.

In this study an attempt has been made to shorten the gap between theory and practice by reducing the amount of mathematical work involved in applying the theory. Since the equations derived are in conformity with measured pressure intensities it is believed that by the use of these equations the effect of concentrated loads can be more accurately predicted than by rule of thumb methods.

⁴ Dr. Raymond D. Mindlin. Proceedings of the International Conference on Soil Mechanics and Foundation Engineering, vol. III, 1936, p. 155.

³ Horizontal Pressures on Retaining Walls Due to Concentrated Surface Loads, by M. G. Spangler. Iowa Engineering Experiment Station Bulletin 140. 1938.

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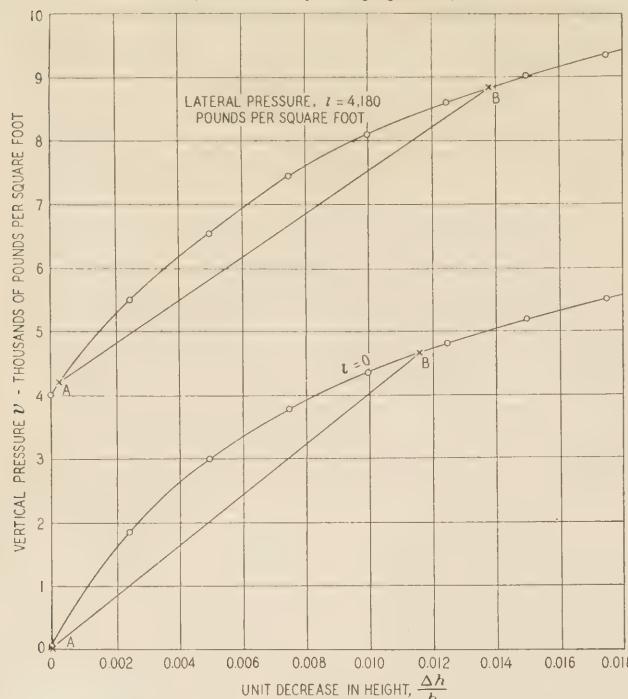


FIGURE 8.—TRIAXIAL TEST DATA FOR FILL SOIL.

lower curve, is $\frac{2,984 - 0}{0.0235 + 0.0001} = 126,000$ pounds per square foot = C . The average modulus C is then taken as $\frac{109,000 + 126,000}{2} = 118,000$ pounds per square foot.

For $\frac{z}{B} = 1.97$, $\mu = \frac{1}{2}$ and $\frac{b}{B} = 0.453$, it is seen from figure 5 that $F = 0.73$. Then, from equation 18, S_L (undersoil) = $\frac{pB}{C}F = \frac{4,662 \times 50.8}{118,000} \times 0.73 = 1.46$ feet.

S_L within the fill itself is now computed and equation 11, taking $K' = 0$, is used for this purpose. Assume that the lateral pressure is zero from 0' to 0, figure 6. This assumption is on the side of safety.

The vertical pressure at 0' is zero whereas at 0 it is equal to p or 4,662 pounds per square foot as already shown. To be consistent with the procedure followed in computing S_L in the undersoil, it is assumed that the point B on each of the two curves, figure 8, corresponds to $v - l = v - 0 = 4,662$ pounds, the maximum $v - l$ in the fill. Figure 8 shows the v versus $\frac{\Delta h}{h}$ characteristics of the compacted fill soil. For the upper curve, figure 8, C is computed as 343,000 pounds per square foot and for the lower curve its value 398,000 pounds per square foot, the average value being 370,000 pounds per square foot. Substituting in equation 11, $S_L(\text{fill}) = \frac{3w}{8C}H^2 = \frac{3 \times 126}{8 \times 370,000} \times 37 \times 37 = 0.18$ foot.

The total settlement due to soil deformation is then $S_L(\text{fill}) + S_L(\text{undersoil}) = 0.18 + 1.46 = 1.64$ feet.

Due to deformation of soil at constant volume, the elevation of the roadway is therefore diminished by 1.64 feet. A considerable part of this settlement would most likely occur during construction of the embankment. To obtain the total settlement, to the settlement S_L must be added the settlement S_c caused by volume changes in the fill and undersoil.

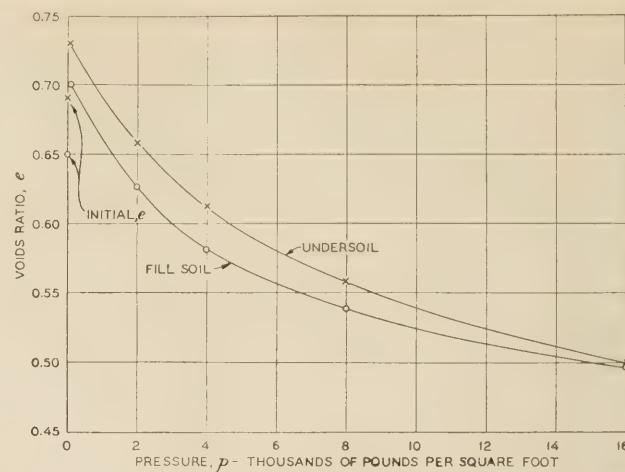


FIGURE 9.—CONSOLIDATION TEST DATA FOR FILL AND FOUNDATION SOILS.

COMPUTATION OF S_c ILLUSTRATED

In figure 9, the voids ratio, e , versus pressure curves are shown for the two soils, fill soil and undersoil. The initial voids ratios of the undisturbed core samples at their natural moisture contents are shown on the e axis in this figure. These are the e values prior to making the consolidation test. For the foundation soil, the initial e is 0.69 and for the fill soil, the initial e is 0.65.

For computing the settlement S_c , the following expression is used,

$$S_c = \frac{e_i - e_f}{1 + e_i} \times D \quad (19)$$

where e_i is the average voids ratio within the thickness D of the soil mass prior to its consolidation and e_f is the average voids ratio within the same soil mass when consolidation is complete. In this problem, D is the height of fill, 37 feet, when computing S_c within the fill and the thickness of the compressible layer of undersoil, 100 feet, when computing S_c in the undersoil.

For the fill, e_i is taken as the initial value, 0.65, the voids ratio of the soil as compacted in the fill. In computing e_f for the fill, it is assumed that the compacted soil expands at the road surface from $e = 0.65$ to $e = 0.70$, as shown in figure 9, this expansion being due to wetting. At the base of the fill, the final value of e corresponds to the fill load of 4,662 pounds per square foot and from figure 9, this value of e is 0.57. Then e_f , the average final value of e is $\frac{0.70 + 0.57}{2} = 0.635$. Then S_c (fill) = $\frac{0.65 - 0.635}{1 + 0.65} \times 37 = 0.34$ foot.

For the undersoil, it is assumed that the clay soil is consolidated under its own weight prior to fill construction. The initial voids ratio of an undisturbed core sample taken at the subgrade surface is seen to be 0.69 from figure 9. At a depth of 100 feet, the vertical pressure due to overburden is $100 \times$ weight per cubic foot of undersoil = $100 \times 128 = 12,800$ pounds per square foot, corresponding to an e of 0.52. Therefore, $e_i = \frac{0.69 + 0.52}{2} = 0.605$, the average e prior to consolidation under the fill load. After complete consolidation under the fill load, e at the point 0, figure 6, corresponds to a pressure of 4,662 pounds per square foot. This value of e , figure 9, is 0.60.

At the bottom of the compressible undersoil layer, point B, figure 6, the final value of e corresponds to a pressure of 12,800 pounds per square foot due to overburden plus the pressure transmitted by the weight of the fill. The latter pressure is the product, pf , where $p=4,662$ pounds per square foot and f is obtained from figure 4. To find f , remember that $\frac{b}{B}=0.453$ and $\frac{z}{B}=1.97$ in this problem. Thus $pf=4,662 \times 0.54=2,517$ pounds per square foot. Then, the total vertical pressure at the lower boundary of the compressible supporting layer of soil is $12,800+2,517=15,317$ pounds per square foot corresponding to an e of 0.50.

Then $e_f = \frac{0.60+0.50}{2} = 0.55$, the average e from top to bottom of the supporting soil subsequent to complete consolidation under the weight of the fill. Using equation 19, S_c (undersoil) $= \frac{0.605-0.550}{1+0.605} \times 100 = 3.43$ feet.

The total S_c is then S_c (fill) + S_c (undersoil) $= 0.34 + 3.43 = 3.77$ feet and total $S_L +$ total $S_c = 1.64 + 3.77 = 5.41$ feet.

SETTLEMENT RECORDS OF EMBANKMENTS NEEDED

The purpose of presenting formulas relating to the bearing capacities and displacements of soils is to enable the engineer to make a reasonably accurate approximation. It would be a serious mistake to rely completely on any mathematical formula derived from theory and assumptions without reference to observations in the field and to practical experience. Where the problem deals with large earth masses, mathematical expressions can at the best only indicate the general trend of phys-

ical occurrences. After many years, the proposed formulas may be modified and accepted or they may be completely rejected depending upon the extent to which, on trial, they have proved to be serviceable. The details in the development of the formulas serve to establish and clarify certain fundamental principles and conceptions which are often of more value than the formulas themselves.

The theoretical expressions for computing settlement caused by lateral displacement, such as equations 11, 16, and 17 would be expected to be more precise for relatively small deformations. As deformations increase in magnitude one or more points within the soil mass may become stressed to a condition characteristic of failure while the soil surrounding these points is stressed below this limit. As soon as a condition of failure is reached anywhere in the soil mass, even though it be reached at only one point, the whole system of stresses throughout the soil in the loaded region changes and the stress-deformation relations expressed by equations 11, 16, and 17 cannot be considered as a true picture of conditions. However, the true limitations of these theoretical expressions which are based on an assumption of a strictly linear relationship between stress and deformation are not known. With certain soils, the relationship may never be linear for stresses of any magnitude whatsoever.

The essential requirement is a correlation of field observations with theoretical and laboratory studies. Computed settlements such as those presented in the example must be checked against observed settlements. Theory may serve as a rough guide or crude yardstick in promoting more thorough and intelligent field studies. This is about the best that can be expected of its use.

¹ Amounts distributed during the calendar year often differ from actual collections because of undistributed funds and lag between accounts of collecting and expending agencies.

² In many States the proceeds of highway user taxes are placed in a common fund from which a distribution is made. The amounts so distributed have been prorated in proportion to the receipts not otherwise dedicated. See following tables.

³ Where reported separately from collection expenses, funds allotted for motor-fuel inspection, administration of motor vehicle department, and regulation of motor vehicles are shown in this column.

⁴ The following amounts for construction and maintenance of county roads under State control are included in all returns for State highway purposes: Delaware, \$279,000; North Carolina, \$7,367,000; Virginia, \$6,554,000; West Virginia, \$2,082,000. In Virginia the 3 counties whose roads are not under State control received \$248,000.

⁵ Reimbursement to counties and local units of government for amounts spent on roads now on State system.

⁶ In States indicated by star (*) law provides that these funds may also be used for service of local highway obligations. Amounts so used not reported separately. In Colorado funds may be used on both State and local roads.

⁷ This column shows specific allotments for city streets. Where reported separately, funds allotted for urban extensions of State highway system are included in allotments for State highways.

⁸ To State general funds, except as follows: Louisiana, 1 cent of tax to parishes; Wisconsin, payment to towns, cities, and villages in lieu of personal property tax formerly imposed on motor vehicles. Allocations to local general funds may have been used in part for highways, but such amounts not reported.

⁹ For the following purposes: Arizona, irrigation engineering expenses; Delaware, State parks, CCC ditching, etc.; Florida, aviation; Louisiana and Massachusetts, harbor improvement; New Jersey, debt service on institutional construction bonds, \$268,000, department of commerce and navigation, \$166,000, other departments, \$65,000; North Carolina, State probation and parole commissions; Pennsylvania, aviation, \$171,000, cooperative work other departments, \$34,000; Tennessee, debt service on nonhighway bonds; Vermont, debt service on nonhighway portion of flood relief bonds; Virginia, aviation.

¹⁰ Includes debt service charges on emergency relief bond issues, prorated in proportion to use of proceeds for State highways, local roads and streets, and nonhighway purposes.

¹¹ Paid out of motor-vehicle revenue, \$6,000. See following table.

¹² Debt service on highway relief funds, a State obligation incurred for improvement of local roads.

¹³ Originally appropriated for relief but later transferred by legislative action to State general fund.

¹⁴ Appropriations for highway purposes out of State general fund have been credited against payments of motor-fuel tax and motor-vehicle revenues to the general fund and prorated in proportion to net receipts from highway user taxes not otherwise dedicated.

¹⁵ Included in cost of collecting motor-vehicle revenue. See following table.

¹⁶ Tax of \$641,000 on nonmotor-vehicle fuels not included.

¹⁷ Expenditures for highway purposes have been credited against payments of motor-fuel tax and motor vehicle revenues to the State general fund and prorated in proportion to net receipts from highway user taxes not otherwise dedicated.

¹⁸ Paid out of general revenue. Amount not reported.

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⁹ To State general funds except in the following States: Alabama, county and municipal general funds; California, general funds of counties and cities, \$3,776,000; New Mexico, county general funds, \$307,000; Wisconsin, towns, cities, and villages in lieu of personal property taxes formerly imposed on motor vehicles, but allocations to county and local general funds may have been used in part for highways, but \$1,370,000. Allocations to county and local general funds not reported.

¹⁰ For the following purposes: Delaware, State parks, CCC, ditching, etc.; Massachusetts, harbor improvement; New Jersey, debt service on institutional construction bonds, \$231,000; Department of Commerce and Navigation, \$162,000, other departments, \$53,000; Ohio, care of indigent persons injured in motor-vehicle accidents, \$73,000; cooperative work other departments, \$14,000.

¹¹ Includes debt service charges on emergency relief bond issues, prorated in proportion to use of proceeds for State highways, local roads and streets, and nonhighway purposes.

¹² Debt service on highway relief bonds, a State obligation incurred for improvement of local roads.

¹³ Appropriations for highway purposes out of State general fund have been credited against payments of motor-fuel tax and motor-vehicle revenues to the general fund and prorated in proportion to net receipts from highway user taxes not otherwise dedicated.

¹⁴ Due to change in registration year from Jan. 1 to April 1.

¹⁵ Expenditures for highway purposes have been credited against payments of motor-fuel tax and motor-vehicle revenues to the State General Fund and prorated in proportion to net receipts from highway user taxes not otherwise dedicated.

¹ Amounts distributed during the calendar year often differ from actual collections because of undistributed funds and lag between accounts of collecting and expending agencies.

² In many States the proceeds of highway user taxes are placed in a common fund from which a distribution is made. The amounts so distributed have been prorated in proportion to the receipts not otherwise dedicated. See preceding and following tables.

³ Collection expenses in many States include service charges deducted by county and local collectors.

⁴ Where reported separately from collection expenses, funds allotted for collection of motor-fuel tax, payments to auto theft fund, and miscellaneous expenses of motor-vehicle regulation are shown in this column.

⁵ The following amounts for construction and maintenance of county roads under State control are included in allotments for State highway purposes: Delaware, \$91,000; North Carolina, \$2,093,000; West Virginia, \$68,000.

⁶ Reimbursement to counties and local units of government for amounts spent on roads now on State system.

⁷ In States indicated by star (*) law provides that these funds may also be used for service of local highway obligations. Amounts so used not reported separately. In Colorado funds may be used on both state and local roads.

⁸ This column shows specific allotments for city streets. Where reported separately funds allotted for urban extensions of State highway system are included in allotments for State highway purposes.

¹ Includes receipts from motor-fuel taxes, motor-vehicle fees and fines, and special imposts on motor vehicles operated for hire (motor-carrier taxes). See preceding tables, which give distribution of receipts separately.

² Amounts distributed during the calendar year often differ from actual collections because of undistributed funds and lag between accounts of collecting and expending agencies.

³ Includes expenses of collection and administration of motor-fuel tax, motor-vehicle fees, and motor-carrier taxes, and miscellaneous expenses of motor-vehicle regulation.

⁴ The following amounts for construction and maintenance of county roads under State control are included in allotments for State highway purposes: Delaware, \$370,000; North Carolina, \$8,566,000; Virginia, \$8,554,000; West Virginia, \$2,730,000. In Virginia, the 3 counties whose roads are not under State control received \$298,000 from the State motor-fuel tax.

⁵ Reimbursement to counties and local units of government for amounts spent on roads now on State system.

⁶ In States indicated by star (*) law provides that these funds may also be used for service of local highway obligations. Amounts so used not reported separately. In Colorado funds may be used on both State and local roads.

⁷ This column shows specific allotments for city streets. Where reported separately, funds allotted for urban extensions of State highway system are included in allotments for State highway purposes.

⁸ To State general funds except in the following States: Alabama, county and municipal general funds;

California, general funds of counties and cities, \$3,776,000; Louisiana, parish general funds, \$2,723,000;

New Mexico, county general funds, \$307,000; Wisconsin, towns, cities, and villages in hen of personal

property tax formerly imposed on motor vehicles, \$3,704,000. Allocations to local general funds may have been used in part for highways, but such amounts not reported.

⁹ For the following purposes: Arizona, irrigation engineering expenses; Delaware, State parks, C. C. C. ditching, etc.; Florida, aviation; Louisiana and Massachusetts, harbor improvement; New Jersey, debt service on institutional construction bonds, \$327,000, department of commerce and navigation, \$328,000, other departments, \$108,000; North Carolina, State probation and parole commissions; Ohio, care of indigent persons injured in motor-vehicle accidents; Pennsylvania, aviation, \$244,000, cooperative work other departments, \$48,000; Tennessee, debt service on nonhighway bonds; Vermont, debt service on nonhighway portion of flood relief bonds; Virginia, aviation.

¹⁰ Includes debt service charges on emergency relief bond issues, prorated in proportion to use of proceeds for State highways, local roads and streets, and nonhighway purposes.

¹¹ Debt service on highway relief bonds, a State obligation incurred for improvement of local roads.

¹² Originally appropriated for relief, but later transferred by legislative action to State general fund.

¹³ Appropriations for highway purposes out of State general fund have been credited against payments of motor-fuel tax and motor-vehicle revenues to the general fund and prorated in proportion to net receipts from highway user taxes not otherwise dedicated.

¹⁴ Due to change in registration year from January 1 to April 1.

¹⁵ Expenditures for highway purposes have been credited against payments of motor-fuel tax and motor-vehicle revenues to the State general fund and prorated in proportion to net receipts from highway-user taxes not otherwise dedicated.

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